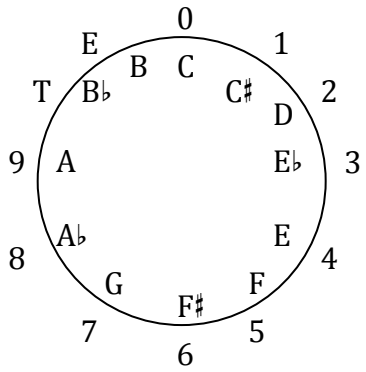
Set Theory & Serialism

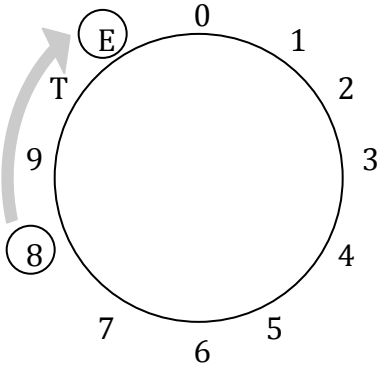
Pitch class

- Group of pitches that share a name
- Includes all notes that are enharmonically equivalent (for instance, C♯ and D♭) and also octave equivalent (middle C = low C = C)
- Only 12 pitch classes exist
- Typically expressed using integers, which can be arranged on a clock face



Unordered pitch class intervals

- The shortest distance between two pitch classes measured in half steps
- It doesn't matter which note you start from, or whether you go clockwise or counterclockwise—just take the shortest route!
- Your answer will always be a positive integer between 0 and 6



The shortest route is clockwise from 8 to E:
8 → E = 3

E → 8 = 9, which is not the shortest route (and 9 > 6)

Interval Class

- An unordered pitch class interval can also be called an **interval class**
- Each interval class includes many different intervals
    - Interval class 3 includes minor thirds (3 semitones), major sixths (9 semitones), minor tenths (15 semitones), etc.
    - Why is this true?
        - Octave equivalence: any interval larger than an octave are equivalent to intervals within the octave ("interval class 3" includes any combination of one or more octaves + 3 half steps)
        - Further, pitch classes include inversional equivalence (an interval is equivalent to its complement, so 9 half steps = 3 half steps). This is why all interval classes are an integer between 0 and 6.
        - Thus, interval class 3 includes any interval that can be reduced to 3 or 9 in the mod12 universe.

Interval Class Vector

- Intervals are very important to atonal music. Without a sense of key or harmonic motion, interval content provides much of the character or "flavor" of an atonal composition (or a portion of a composition).
    - For instance, a group of pitches that include many minor thirds will sound very different from a group of pitches saturated by whole steps or tritons.
- An interval class vector allows us to visually represent intervallic content in a clear and efficient way.
- To write an interval class vector, first tally every instance of each ic (excluding 0)
    - The set (0, 4, 7, 10), which represents a Mm7 chord, includes
        - 0 instances of ic 1
        - 1 instance of ic 2 (from 10 to 0)
        - 2 instances of ic 3 (from 4 to 7 and 7 to 10)
        - 1 instance of ic 4 (from 0 to 4)
        - 1 instance of ic 5 (from 7 to 0
        - 1 instance of ic 6 (from 4 to 6)
- An interval class vector is presented as a string of six numbers with no spaces
    - The ic vector for the set (0, 4, 7, 10) is **012111**, which includes the tallies for each ic class as shown above

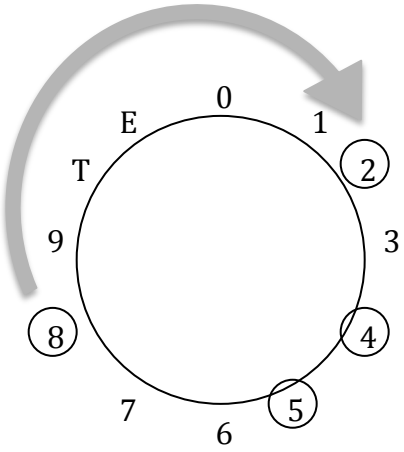
Pitch-Class Sets

- Unordered collection of pcs; functions like a motive

Normal Form

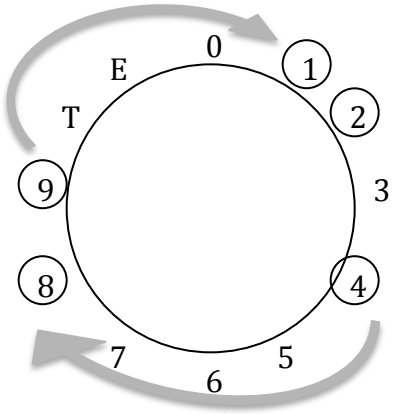
- The most compressed way of writing a pc set
- Shown in brackets
- How to find normal form:

1. Circle the pcs on the clockface
2. Look for the biggest gap. If there is one big, start with the pc at the end of the big gap, moving clockwise (this will keep the largest gap between the last and first integer).



In the set 2,4,5,8 the largest gap is between 8 and 2. So, the normal form begins on 2: [2,4,5,8]

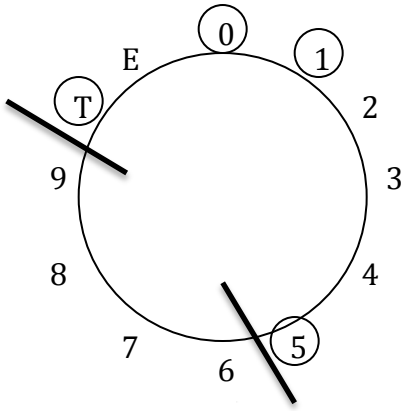
3. If there is a tie, look at next interval (first note to next-to-last note); smaller interval is normal form.



In the set 1,2,4,8,9 there are two three-semitone gaps, suggesting two orderings:
1,2,4,8,9 and 8,9,1,2,4
Now, we need to look at the interval from the first note to the next-to-last note.
1,2,4,8,9: 1-8=7
8,9,1,2,4: 8-2=6* (smaller than 7)
So, [8,9,1,2,4] is normal form.

Prime Form

- Prime form is a way to compare all of the inversions and transpositions of a set to find the "most normal" (most packed to the left) ordering.
    - Normal form only compares transpositions.
- We will use the clockface to find prime form:
  1. Circle the pcs on the clockface.
  2. Find the biggest gap. Compare clockwise and counterclockwise paths. Which has a smaller span between the first and next-to-last elements?
  3. Transpose your solution to start on 0, and write in parentheses without commas.
- Find the prime form for the set Bb, C, C#, F:



1. Biggest gap is between 5 and T. Two potential orderings:
T015
510T
2. Moving clockwise will give us a more compact ordering, with a smaller span between the first and next-to-last element.
T015: T→1=3
510T: 5→0=5
3. Transpose [T, 0, 1, 5] to start on 0:
(0237)

Transposition (T_n)

- Sets can be transposed by adding a consistent number (n) to each pc
- n=the interval of transposition measured in half steps
- To transpose the set [8,9,1,2,4] by 7 half steps, simply add 7 to each pc, mod12
    - Add 7 and you get 15, 16, 8, 9, E
    - Adjust for mod12 and you get [3,4,8,9,E]
- When you transpose a set that is in normal form, your transposition will also be in normal form.
- To tell if one set is a transposition of another, put them both in normal form and subtract the elements of one set from the elements of the other. If the answer is the same for each element, the set is a transposition.
    - Is the set [8,9,1,2,4] a transposition of [3,4,8,9,E]?

      8 9 1 2 4 -
      3 4 8 9 E
      5 5 5 5 5

      Yes, the two sets are related by transposition.

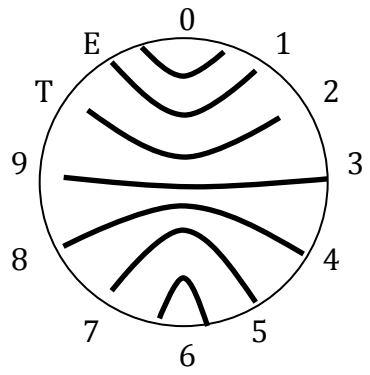
- Is the set [6,7,T,0,1] a transposition of [3,4,8,9,E]?

  6 7 T 0 1 -
  3 4 8 9 E
  3 3 **2** 3 **2**

  No, the two sets are not related by transposition.

Inversion (T_nI)

- Inversion is a compound operation. First, you invert around 0, then you transpose by *n*.
- When you invert around 0, each pc is replaced by its complement.



- To perform the operation T_8I on the set 67T01:
  1. Replace each pc with its complement: 6420E
  2. Transpose the new set by adding 8 to each element: 20T87
  3. Put the set in normal order: [7,8,T,0,2]

Serialism

- Some music uses all 12 pcs as a motive. In this music, the ordering of the pcs is of paramount importance.
- The first statement of the aggregate (all 12 pcs) is the "row." This row can be present in multiple different formats, or row forms. We label each row form with the first pc of that row.
  - Prime (P)
    - The row's original ordering—the first way the row is presented
  - Retrograde (R)
    - Reverse order of P (order numbers reversed)
  - Inversion (I)
    - Intervals inverted (replaced by their complement)
  - Retrograde inversion (RI)
    - Reverse of the inversion (so invert first, then retrograde)

- We can show all possible permutations of a row using a matrix.

I ↓

P →												← R
	0	6	8	5	7	E	4	3	9	T	1	2
	6	0	2	E	1	5	T	9	3	4	7	8
	4	T	0	9	E	3	8	7	1	2	5	6
	7	1	3	0	2	6	E	T	4	5	8	9
	5	E	1	T	0	4	9	8	2	3	6	7
	1	7	9	6	8	0	5	4	T	E	2	3
	8	2	4	1	3	7	0	E	5	6	9	T
	9	3	5	2	4	8	1	0	6	7	T	E
	3	9	E	8	T	2	7	6	0	1	4	5
	2	8	T	7	9	1	6	5	E	0	3	4
	E	5	7	4	6	T	3	2	8	9	0	1
	T	4	6	3	5	9	2	1	7	8	E	0

RI ↑

- If you don't have a matrix to hand, you can still determine row forms.
- First, determine the order of intervals between members of the P form

P0: 0 6 8 5 7 E 4 3 9 T 1 2
6 2 9 2 4 5 E 6 1 3 1

- R forms: complement of P, in reverse

R2: 2 1 T 9 3 4 E 7 5 8 6 0
E 9 E 6 1 7 8 T 3 T 6

- I forms: complement of P

I0: 0 6 4 7 5 1 8 9 3 2 E T
6 T 3 T 8 7 1 6 E 9 E

- RI forms: reverse of P

RIT: T E 2 3 9 8 1 5 7 4 6 0
1 3 1 6 E 5 4 2 9 2 6

For more information on this topic see *Musician's Guide* pp. 715–751, 768–771.